2020

(Held in 2021)

PHYSICS

(Major)

Paper : 5.1

Full Marks : 42

Time : 2 hours

The figures in the margin indicate full marks for the questions

Both the Group contains questions of Mathematical Methods and Classical Mechanics

GROUP—A

(*Marks* : 21)

1. Answer the following questions : 1×2=2

(a) What is the argument of $-3i$?

(b) State the principle of virtual work.

2. Answer the following questions : 2×2=4

(a) Obtain the modulus of the complex number $\frac{1-i}{1+i}$.

(b) Show that in a central force field the angular momentum of a particle is conserved.

3. Answer any *three* questions from the following : 5×3=15

(a) State and prove Cauchy's integral theorem.

(b) Set up the Lagrangian for a simple pendulum and hence obtain equation describing its motion.

(c) Show that

$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^2} = \pi$$

(d) Starting from the Hamilton's principle, deduce the Lagrange's equations of motion.

(e) Show that a two-body central force problem can be reduced to one-body problem.

GROUP—B

(*Marks* : 21)

4. Answer any *three* questions from the following : 7×3=21

(a) A particle moves on a curve $r^n = a^n \cos n\theta$ under the influence of a central force. Find the force law. If a negatively charged particle moves under coulomb force of the nucleus, deduce the nature of the orbit and periodic time. 3+2+2=7

(b) Separate $\sin(x+iy)$ into real and imaginary parts, x and y being both real. Show that

$$|\sin(x+iy)|^2 = \sin^2 x + \frac{1}{4}(e^y - e^{-y})^2$$

4+3=7

(c) Two particles of mass m and M are joined by a rod of fixed length l. The particle of mass M is constrained to move along the vertical axis Y, while the other mass m along the horizontal X-axis. Apply D'Alembert's principle to find equation of motion of the system. A particle is moving on an ellipsoid under influence of gravity. Name the involved constraint. 6+1=7

(d) Calculate the residues of $f(z) = \frac{z^2}{(1+z^2)^2}$ and evaluate $\int_0^{\infty} \frac{x^2 dx}{(1+x^2)^2}$. 3+4=7

(e) Find the first three terms of Taylor expansion of $f(z) = \frac{1}{z^2+4}$ about $z = -i$ and give the region of convergence. 6+1=7

★★★