

Total number of printed pages-7

3 (Sem-4/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY-HC-4016

(Mathematical Physics-III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Give short answers to the following questions : 1×7=7

(a) Find the principal value of i^i .

(b) Define a multiply connected region in complex plane.

(c) Find the value of $L^{-1}\left\{\frac{1}{s(s-a)}\right\}$ for $s > a$.

Contd.

- (d) What does the equation $|z - i| = 2$ represent ?
- (e) State convolution theorem of Fourier transform.
- (f) Write the transformation rule for a covariant tensor of rank two.
- (g) Plot the number $e^{(1+i\frac{\pi}{4})}$.

2. Answer the following questions: $2 \times 4 = 8$

- (a) Define simple pole and essential singularity.
- (b) Establish the shifting property of Fourier transform.
- (c) Find inverse Laplace transform of $t^{-\frac{1}{2}}$.

- (d) Show that the number of independent components of a skew-symmetric tensor of rank 2 in n -dimensional space is

$$\frac{n(n-1)}{2}.$$

3. Answer **any three** questions of the following: 5×3=15

- (a) Check the analyticity and hence find derivative of the function $f(z) = \sin z$.

$$3+2=5$$

- (b) Find the value of the integral

$$\int_0^{1+i} (x - y - ix^2) dz \text{ along real axis from}$$

$z = 0$ to $z = 1$ and then along the line parallel to imaginary axis from $z = 1$ to $z = 1+i$.

- (c) Find the Fourier sine transform of a function defined by

$$f(t) = \begin{cases} \cos \alpha t, & 0 \leq t \leq \frac{\pi}{\alpha} \\ 0, & t > \frac{\pi}{\alpha} \end{cases}$$

- (d) Evaluate :

$$L^{-1} \left\{ \frac{(s+1)}{s^2 (s+2)^3} \right\}$$

- (e) Define Levi-Civita symbol in three dimensional space. Show that

$$(\bar{A} \times \bar{B})_i = \varepsilon_{ijk} A_j B_k \quad 2+3=5$$

4. Answer the following questions : **(any three)**

$$10 \times 3 = 30$$

- (a) (i) Prove Cauchy-Riemann conditions for analytical functions. What is the sufficient condition for a function to be analytic ? 4+1=5

(ii) Show that $|z_1 + z_2| \leq |z_1| + |z_2|$

3

(iii) Give Laurent series expansion for function $f(z)$.

2

(b) (i) What are symmetric and antisymmetric tensors? Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other antisymmetric in a pair of co-variant or contravariant indices.

2+3=5

(ii) What is Kronecker delta? Prove that Kronecker delta is a mixed tensor of second rank.

1+4=5

(c) (i) Define Laplace transform of a function $F(t)$. Show that

$$L(1) = \frac{1}{s}, \quad s > 0 \quad \text{and}$$

$$L(e^{kt}) = \frac{1}{s-k}, \quad s > k \quad 1+2+2=5$$

- (ii) Find the inverse Laplace transform of

$$\frac{6}{2s-3} - \frac{3+4s}{9s^2-16} + \frac{8-6s}{16s^2+9} \quad 5$$

- (d) Find the Fourier transform of

$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

Hence, evaluate :

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$$

- (e) Evaluate *any two* of the following integrals : 5×2=10

(i) $\int_{-\infty}^{+\infty} \frac{\sin x}{x} dx$

(ii) $\int_0^{\infty} \frac{dx}{x^2+1}$

$$(iii) \int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$$

(f) (i) The Laplace transform of $\sin 3t = \frac{3}{s^2 + 9}$ and the Laplace

transform of $\cos 5t = \frac{s}{s^2 + 25}$.

Find the Laplace transform of $5 \sin 3t + 3 \cos 5t$ using linearity property of Laplace transform. 5

(ii) Find the inverse Laplace transform of $4s + 5 / (s - 1)^2 (s - 2)$. 5
